

Time-Reversal-Symmetry-Broken Superconductivity Induced by Frustrated Inter-Component Couplings

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Superconductivity is associated with spontaneously broken gauge symmetry. In some exotic superconductors the time-reversal symmetry is broken as well, accompanied with internal magnetic field. A time-reversal symmetry broken (TRSB) superconductivity without internal magnetic field involved can be induced by frustrated inter-component couplings, which becomes a realistic issue recently due to the discovery of iron-pnictide superconductors. Here we derive stability condition for this novel TRSB state using the Ginzburg-Landau (GL) theory. We find that there are multiple divergent length scales, and that this novel superconductivity cannot be categorized by the GL number into type I or type II. We reveal that the critical Josephson current of a constriction junction between two bulk superconductors of different chiralities is suppressed significantly from that for same chirality. This effect provides a unique way to verify experimentally this brand new superconductivity.

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Spontaneous symmetry breaking is one of the most fundamental concepts of modern physics, which plays a central role in particle physics, condensed matter physics, and even in our understanding into the birth of universe. Broken gauge symmetry is known as the hallmark of superconductivity [1, 2]. Simultaneous breaking of gauge symmetry and time-reversal symmetry has attracted considerable attentions [3], with the triplet superconductivity in Sr_2RuO_4 as an icon [4].

Possibility of time-reversal-symmetry-broken (TRSB) superconductivity in multi-component system was discussed before in the context of conventional mechanism for exotic superconductivity [5]. Interests in this novel phenomenon are renewed [6, 7] by the discovery of iron-based superconductors [8]. Previous works revealed that frustrated inter-component couplings induce intrinsically complex order parameters, and thus the TRSB superconductivity [5–7], primarily based on a special situation with all components equivalent (the *isotropic* system).

It was also discussed that a junction structure between superconductors of two components and single component can exhibit a similar TRSB situation [9]. Possibility of TRSB state was discussed in iron-based superconductors with coexisting s- and d-wave order parameters [10]. A novel dynamic mode was proposed for three-component, time-reversal symmetry reserved (TRSR) superconductivity [11]. All these make the physics of multi-component superconductivity very rich.

In the present work, we consider an *anisotropic* system with frustrated inter-component couplings. For simplicity, each component is considered as s-wave and well described by the BCS theory [1]. We adopt the GL approach [12], powerful for discussions on stability of states and thus phase transition, symmetry breaking, as well as magnetic responses of superconductivity, which are the primary concerns of the present study. Benefitting from the simplicity of the GL theory, the condition for stable TRSB state are derived explicitly. It is also found that

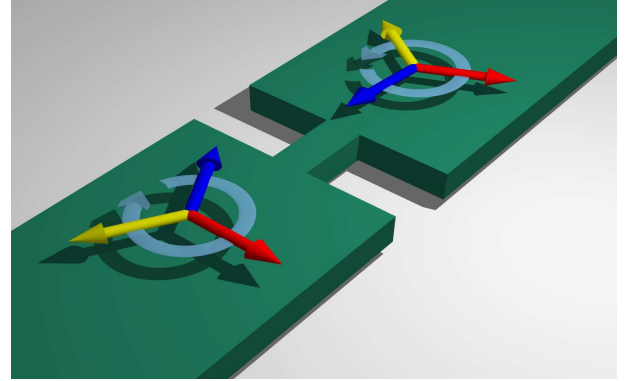


FIG. 1: TRSB states characterized by complex order parameters and opposite chiralities of a three-component superconductor in a Josephson junction of constriction structure.

there are multiple divergent length scales, and that this novel superconductivity cannot be categorized by the GL number into type I and type II. We reveal that the critical Josephson current of a constriction junction between two bulk superconductors of different chiralities (see Fig. 1) is suppressed significantly from that for same chirality. A standard Josephson junction measurement can provide smoking gun evidence for this brand new superconductivity.

The density of GL free-energy functional for a multi-component superconductor is:

$$f = \sum_j \left[a_j |\psi_j|^2 + \frac{1}{2} b_j |\psi_j|^4 + \frac{1}{2m_j} \left| \left(\frac{\hbar}{i} \nabla - \frac{2e}{c} \mathbf{A} \right) \psi_j \right|^2 \right] - \sum_{j < k} \gamma_{jk} (\psi_j \psi_k^* + \text{c.c.}) + \frac{1}{8\pi} (\nabla \times \mathbf{A})^2, \quad (1)$$

where summations running over the number of components are understood, and all quantities are conventionally defined (see [5, 7, 12, 13]); $a_j(T_{cj}) = 0$ specify the transition points for individual components. To be spe-

cific, we discuss here a system with three components, and extension of the following discussions to more components is straightforward. It is noticed that the physics addressed below remains unchanged even when other possible terms (up to the quartic order of order parameters) are included in Eq.(1).

For $\gamma_{12}\gamma_{23}\gamma_{13} > 0$, the system behaves basically as a single-band superconductivity close to the critical point of the composite system. Hereafter, we focus on the case $\gamma_{12}\gamma_{23}\gamma_{13} < 0$, and treat a system with all γ_{jk} 's negative, noticing that a simple gauge transformation in GL free energy (1) links it to other possible cases. The situation treated here may be realized in over-doped iron pnictides [6].

Around the critical point, the GL equations in absence of magnetic field can be linearized as

$$\begin{bmatrix} a_1 & -\gamma_{12} & -\gamma_{13} \\ -\gamma_{12} & a_2 & -\gamma_{23} \\ -\gamma_{13} & -\gamma_{23} & a_3 \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad (2)$$

or in a vector form $\mathbf{Q} \cdot \Psi = \mathbf{0}$ with coupling matrix \mathbf{Q} . The critical point of the composite superconductivity T_c is given by the highest temperature where the determinant of \mathbf{Q} becomes zero, i.e.

$$a_1 a_2 a_3 - 2\gamma_{12}\gamma_{23}\gamma_{13} - a_1\gamma_{23}^2 - a_2\gamma_{13}^2 - a_3\gamma_{12}^2 = 0, \quad (3)$$

where temperature dependence of the quantities is understood (see Fig. 2). From Sylvester's criterion [14], one has $a_j > 0$ and $a_j a_k - \gamma_{jk}^2 \geq 0$ at T_c , namely the critical point of the composite superconductivity is above any single components, and not below any of the two-component ones, in spite of negative couplings [15].

If Eq.(3) has a single root at $T = T_c$ (see Fig. 2), or equivalently there are two independent vectors in the coupling matrix \mathbf{Q} , the ratios among the order parameters given by the Cramer's rules for the components of the matrix \mathbf{Q} should be real [14]. The order parameters of three-component superconductivity that minimize the GL free energy (1) can then be taken as real numbers, apart from a common phase factor, same as single- and two-component superconductivity.

Equation (3) has a doubly degenerated root [16] (see Fig. 2), or equivalently there is only one independent vector in \mathbf{Q} , when

$$a_1 a_2 - \gamma_{12}^2 = 0, \quad a_1 a_3 - \gamma_{13}^2 = 0, \quad a_2 a_3 - \gamma_{23}^2 = 0, \quad (4)$$

at T_c , namely $T_c = T_{c12} = T_{c13} = T_{c23}$. The single independent vector in the coupling matrix \mathbf{Q} leaves the room for complex order parameters in Eq.(2) despite that all the parameters in \mathbf{Q} are real. Equation (4) is the first condition for a TRSB state specified by complex order parameters.

It is easy to see that relations in Eq. (4), as well as the associated ones $a_j \gamma_{kl} + \gamma_{jk} \gamma_{jl} = 0$, correspond to single zeros, since $\gamma_{jk} \neq 0$. One finds $(a_k a_l - \gamma_{kl}^2)/(a_j a_l - \gamma_{jl}^2) = (a_k + \gamma_{jk} \gamma_{kl}/\gamma_{jl})/(a_j + \gamma_{jk} \gamma_{jl}/\gamma_{kl}) = (\gamma_{kl}/\gamma_{jl})^2$.

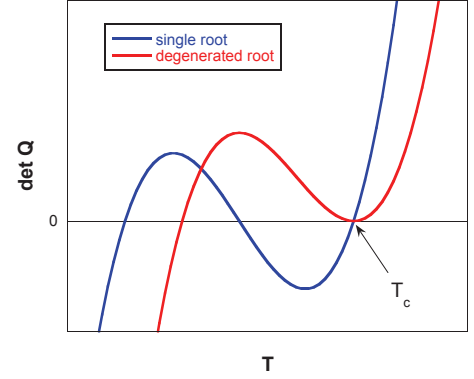


FIG. 2: (color online). Schematic temperature dependence of determinant of the coupling matrix of the linearized GL equations (see Eqs.(2) and (3)). The doubly degenerated root at T_c is a necessary condition for stable TRSB state, whereas the single-root case is associated with a conventional, TRSR state similar to single-component systems.

The order parameters for $T \lesssim T_c$ are given by

$$\begin{bmatrix} a_1 + b_1 |\psi_1|^2 & -\gamma_{12} & -\gamma_{13} \\ -\gamma_{12} & a_2 + b_2 |\psi_2|^2 & -\gamma_{23} \\ -\gamma_{13} & -\gamma_{23} & a_3 + b_3 |\psi_3|^2 \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \quad (5)$$

Putting ψ_1 real as always possible, the imaginary parts in ψ_2 and ψ_3 should obey the relations

$$\begin{bmatrix} a_2 + b_2 |\psi_2|^2 & -\gamma_{23} \\ -\gamma_{23} & a_3 + b_3 |\psi_3|^2 \end{bmatrix} \begin{bmatrix} \text{Im}(\psi_2) \\ \text{Im}(\psi_3) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (6)$$

Therefore, for complex order parameters one has $a_3 b_2 |\psi_2|^2 + a_2 b_3 |\psi_3|^2 \simeq -a_2 a_3 + \gamma_{23}^2$ up to $O(t)$ with $t \equiv (T_c - T)/T_c$. In the same way, one obtains two other similar relations, and thus

$$\begin{bmatrix} 0 & a_3 b_2 & a_2 b_3 \\ a_3 b_1 & 0 & a_1 b_3 \\ a_2 b_1 & a_1 b_2 & 0 \end{bmatrix} \begin{bmatrix} |\psi_1|^2 \\ |\psi_2|^2 \\ |\psi_3|^2 \end{bmatrix} = \begin{bmatrix} -a_2 a_3 + \gamma_{23}^2 \\ -a_1 a_3 + \gamma_{13}^2 \\ -a_1 a_2 + \gamma_{12}^2 \end{bmatrix}. \quad (7)$$

We then arrive at the following temperature dependence of order parameters:

$$|\psi_j|^2 \simeq -(a_j + \gamma_{jk} \gamma_{jl}/\gamma_{kl})/b_j. \quad (8)$$

up to $O(t)$.

The single independent relation in Eq.(2), for example, $a_1 - \gamma_{12}\psi_2/\psi_1 - \gamma_{13}\psi_3/\psi_1 = 0$, is then equivalent to

$$\frac{a_1}{\sqrt{b_1}} + \frac{a_2}{\sqrt{b_2}} e^{i\phi_{21}} + \frac{a_3}{\sqrt{b_3}} e^{i\phi_{31}} = 0 \quad (9)$$

for $T \lesssim T_c$, where ϕ_{21} (ϕ_{31}) is the phase difference between ψ_2 (ψ_3) and ψ_1 . It becomes clear that the condition for a state of complex order parameters to be stable is equivalent to that of a triangle formed by three segments:

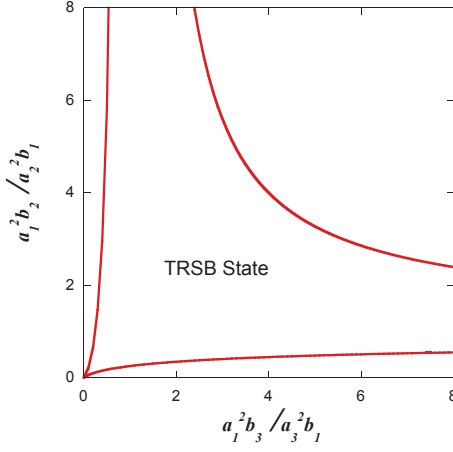


FIG. 3: (color online). Phase diagram for a three-component superconductor with stable TRSB superconductivity at the central part and TRSR one at the corners.

$$\frac{a_j}{\sqrt{b_j}} + \frac{a_k}{\sqrt{b_k}} > \frac{a_l}{\sqrt{b_l}} \quad (10)$$

for $T \lesssim T_c$. A phase diagram is displayed in Fig. 3.

The relations (4) and (10) formulate the full condition for the stability of state with complex order parameters, *i.e.* the TRSB superconductivity. It is clear that the special case with isotropic parameters discussed previously [5–7] satisfies these conditions.

A phase transition at a lower temperature $T_{tr} < T_c$ from a TRSB state to a TRSR state is possible for appropriate temperature dependence of parameters (see Fig. 3), where interesting physics is expected. Although discussions in the GL scheme can be pushed forward, we notice that to treat the two transitions concretely one needs a more microscopic theory.

Next we investigate the coherence length of the state of complex order parameters. In order to demonstrate the novelty of this state, we concentrate for a while on a system of isotropic parameters in Eq.(1) except $m_1 \equiv m$ and $m_2 = m_3 \equiv m'$, the simplest, but non-trivial case allowing analytic treatment. In the bulk, the amplitude of the order parameter is $\psi_0 \equiv |\psi_j| = \sqrt{-(a+\gamma)/b}$, and the phase difference between any of the two components is $2\pi/3$ [5–7]. A local distortion in the first component $\psi_1 = (1 + \delta_1)\psi_0$ causes distortions in the other two components $\psi_2 = (1 + \delta_2)\psi_0 \exp[i(2\pi/3 + \delta_3)]$ and $\psi_3 = (1 + \delta_2)\psi_0 \exp[i(4\pi/3 - \delta_3)]$, with δ_j 's real, as displayed in Fig. 4a. In the one-dimensional case, the GL equation for the variation of ψ_1 , for example, is $a\psi_1 + b\psi_0^2\psi_1 - \gamma(\psi_2 + \psi_3) = (\hbar^2/2m)\partial^2\psi_1/\partial x^2$, which, with two other similar equations, yield

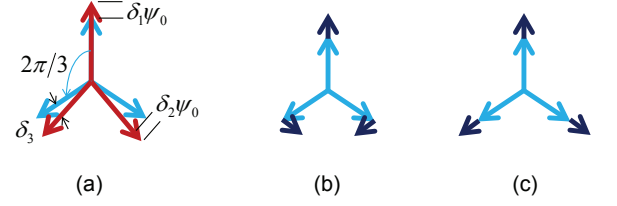


FIG. 4: (color online). Distortions in the complex order parameters in TRSB state of the *isotropic* system (see text). (a) Definitions of distortions in order parameters. (b) and (c) Characteristic distortions of mode-I and model-II respectively, for $m = m'$.

$$\begin{aligned} (a + 3b\psi_0^2)\delta_1 + \gamma\delta_2 + \sqrt{3}\gamma\delta_3 &= \frac{\hbar^2}{2m} \frac{\partial^2 \delta_1}{\partial x^2}, \\ \frac{\gamma}{2}\delta_1 + (a + 3b\psi_0^2 + \frac{\gamma}{2})\delta_2 - \frac{\sqrt{3}\gamma}{2}\delta_3 &= \frac{\hbar^2}{2m'} \frac{\partial^2 \delta_2}{\partial x^2}, \\ \frac{\sqrt{3}\gamma}{2}\delta_1 - \frac{\sqrt{3}\gamma}{2}\delta_2 + (a + b\psi_0^2 - \frac{\gamma}{2})\delta_3 &= \frac{\hbar^2}{2m'} \frac{\partial^2 \delta_3}{\partial x^2}. \end{aligned} \quad (11)$$

The coherence length defined by $\delta_j = A_j \exp(-\sqrt{2}x/\xi)$ at large distance limit is thus determined as

$$\frac{\hbar^2 \xi^{-2}}{-(a+\gamma)m} = \frac{3m'}{2m} \frac{2 + \frac{m'}{m} \pm \sqrt{(\frac{m'}{m})^2 - \frac{4m'}{3m} + \frac{4}{3}}}{1 + 2m'/m}. \quad (12)$$

There are two divergent solutions, and the corresponding characteristic modes are given by $A_2/A_1 = -R/[2(3 - 2R)]$ and $A_3/A_1 = -\sqrt{3}(2 - R)/[2(3 - 2R)]$, where R is defined by the right-hand side of Eq.(12). Mode-I associated with the larger solution, thus giving the coherence length of the system, is specified by $R = 1$, and $A_2/A_1 = -1/2$ and $A_3/A_1 = \sqrt{3}/2$ when $m = m'$ (see Fig. 4b), whereas mode-II by $R = 2$, and $A_2/A_1 = 1$ and $A_3/A_1 = 0$ (see Fig. 4c), with the distortion vectors form an equilateral triangle in both cases. While mode-II at $m = m'$ is the conventional one associated merely with variation of amplitude (see Fig. 4c), known in single- and two-component cases, mode-I is the novel one in which variations of amplitude and phase are coupled (see Fig. 4b), specific to the TRSB state.

Other quantities for the present superconductivity are available straightforwardly, such as the London penetration depth λ

$$\lambda^{-2} = \frac{4\pi(2e)^2}{c^2} (-1 - \gamma_{12}\gamma_{13}/a_1\gamma_{23}) \sum_j a_j/b_j m_j, \quad (13)$$

the thermodynamic critical field H_{tc} from Eq.(8)

$$\frac{H_{tc}^2}{8\pi} = \frac{1}{2} (1 + \gamma_{12}\gamma_{13}/a_1\gamma_{23})^2 \sum_j a_j^2/b_j, \quad (14)$$

and the nucleation field H_n (or H_{c2})

$$H_n^2 = \frac{\phi_0}{2\pi} \frac{\det \mathbf{Q}}{-(a_1 a_2 - \gamma_{12}^2) a_3} \frac{1}{\sum_j \hbar^2 / 2a_j m_j}. \quad (15)$$

It is clear that $H_{tc} \neq \phi_0 / 2\sqrt{2}\pi\xi\lambda$, and $H_n \neq \phi_0 / 2\pi\xi^2$. Consequently, we are led to conclude that this superconductivity cannot be categorized by the GL number $\kappa \equiv \lambda/\xi$ into type I or type II [12].

Because of the existence of two divergent solutions of Eq.(12), a vortex can exhibit different core sizes [12] for the three components even close to T_c . This may cause a long-range attractive and short-range repulsive interaction between two vortices and thus exotic vortex states, a possibility discussed previously in two-component case [17].

Finally let us investigate the Josephson current between two bulks linked by a short, narrow constriction in otherwise continuous superconducting material, as displayed in Fig. 1. The length of the constriction is much shorter than the coherence length $l \ll \xi$. Suppose that the two bulks exhibit opposite chiralities by chance in a cooling process, with ψ_{jL} and ψ_{jR} the wave functions at the left and right bulk respectively, where $|\psi_{jL}| = |\psi_{jR}| \equiv \psi_{j0}$, $\psi_{1R}/\psi_{1L} = e^{i\varphi}$, $\psi_{2L}/\psi_{1L} = e^{i\phi_{21}}\psi_{20}/\psi_{10}$, $\psi_{3L}/\psi_{1L} = e^{i\phi_{31}}\psi_{30}/\psi_{10}$, $\psi_{2R}/\psi_{1R} = e^{i(2\pi-\phi_{21})}\psi_{20}/\psi_{10}$, and $\psi_{3R}/\psi_{1R} = e^{i(2\pi-\phi_{31})}\psi_{30}/\psi_{10}$. The order parameters on the bridge $\psi_j(x) \equiv f_j(x)\psi_{jL}$ are determined by $a_1 f_1 + b_1 \psi_{10}^2 |f_1|^2 f_1 - \gamma_{12} f_2 \psi_{2L}/\psi_{1L} - \gamma_{13} f_3 \psi_{3L}/\psi_{1L} = (\hbar^2/2m_1)\partial^2 f_1/\partial x^2$, and two other similar equations. Following the idea developed for conventional single-component superconductors [12, 18], one gets the linear solutions $f_1(x) = (1-x/l) + e^{i\varphi}x/l$, $f_2(x) = (1-x/l) + e^{i(2\pi-2\phi_{21}+\varphi)}x/l$ and $f_3(x) = (1-x/l) + e^{i(2\pi-2\phi_{31}+\varphi)}x/l$, with $x=0, l$ at the left and right ends of the link, since $(\hbar^2/2m_j)\partial^2 f_j/\partial x^2 = O(\psi_{j0}^2) \propto \xi^{-2} \sim 0$ for $l \ll \xi$.

The Josephson current between the two bulks is then

[12, 18]

$$I = i_1 \sin \varphi + i_2 \sin(\varphi - 2\phi_{21}) + i_3 \sin(\varphi - 2\phi_{31}), \quad (16)$$

with $i_j \equiv e\hbar^2\psi_{j0}^2/m_j$. We arrive at the critical Josephson current

$$I_c = \sqrt{\sum_j i_j^2 + 2 \sum_{j < k} i_j i_k \cos 2\phi_{jk}}, \quad (17)$$

which equals zero in the isotropic case. On the other hand, when the two bulks share the same chirality, one has [12, 18] $I = (i_1 + i_2 + i_3) \sin \varphi$ in contrast to Eq.(16). Therefore, because of the interference among the components, the critical Josephson current between two bulk superconductors of different chiralities is much smaller than the one for same chirality.

In experiment based on the constriction structure shown schematically in Fig. 1, the two bulk superconductors can show either the same chirality or the opposite ones by chance in repeated cooling processes. Then the standard measurement of critical Josephson current will give two sets of values, quite different from each other as discussed above, which gives a smoking gun evidence for this novel superconductivity.

To finish we notice that although the GL approach is, in principle, justified only close to the critical point, the phenomena revealed in present are expected for the whole temperature regime.

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